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Neutrino Trapping:

There are several processes that contribute to the coupling between neutrinos and matter. The most important ones are:

(1) Scattering between neutrinos and free nucleons:

$\nu + n \to \nu + n$, $\nu + p \to \nu + p$

These happen via neutral-current weak interactions.

(2) Scattering between neutrinos and nucleons inside a nucleus:

$\nu + (A, Z) \to \nu + (A, Z)$

These also happen via neutral-current weak interactions. However the cross section for these scatterings receives an enhancement factor of A^2 because of coherent interactions between a neutrino and A nucleons at energies of interest. As a result, they become dominant processes at sufficiently high densities.

(3) Absorption of neutrinos via charged-current weak interactions:

$\nu + p \rightarrow e^{+} + n$, $\nu + n \rightarrow e^{-} + p$

(4) Scattering of neutrinos off electrons via neutral-current weak interactions:

$\nu + e^{-} \rightarrow \nu + e^{-}$

Process (1), (2) are elastic for neutrino energies $E_\nu \ll m_N c^2$ (as is the case in supernova). On the other hand, (4) is not elastic and results in neutrino energy loss. It is thus quite important in thermalizing the neutrinos.

The scattering cross section can be computed for all of these processes. The partial cross section for neutrino-nucleus cross section is given by:

$$\frac{d\sigma}{d\Omega} = \sigma_0 \left(\frac{A^2}{64\pi}\right)\left(\frac{E_\nu}{m_e c^2}\right)^2 (1+\cos\theta) \qquad (E_\nu \ll m_N c^2)$$

$$\sigma_0 = \frac{4}{\pi}\frac{G_F^2 m_e^2}{\hbar^4} = 1.7\times 10^{-44}\ cm^2$$

Here G_F is the Fermi Constant. Note the smallness of σ_0 as compared to the Thomson scattering cross section $\sigma_T \approx \frac{2}{3}\times 10^{-24}\ cm^2$, which is due to the interaction being weak.

Forward scatterings $\theta \simeq 0$ do not affect neutrinos. Excluding these scatterings, we find the total cross section for neutrino nucleus scattering to be:

$$\boxed{\sigma_{\nu A} = \sigma_0 \left(\frac{A^2}{24}\right)\left(\frac{E_\nu}{m_e c^2}\right)^2}$$

For neutrino-nucleon scattering we have:

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2}{(2\pi)^2}\frac{1}{4}E_\nu^2\left[(1+\cos\theta) + 3g^2\left(1-\frac{1}{3}\cos\theta\right)\right]$$

Here $g \simeq 1.25$ is a constant related to the weak interactions. The total cross section (excluding forward scatterings) is found to be:

$$\boxed{\sigma_{\nu N} = \left(\frac{\sigma_0}{24}\right)\left(\frac{E_\nu}{m_e c^2}\right)^2 (1+5g^2)}$$

The overall cross section for scattering of neutrinos is:

$$\sigma_\nu = \sigma_{\nu A} + \sigma_{\nu N}$$

The mean free path of neutrinos λ_ν will be:

$$\lambda_\nu = \frac{1}{n\sigma_\nu} \simeq (1.0\times10^6\,cm)\, \rho_{12}^{-1}\left(\frac{\langle E_\nu\rangle}{10\,MeV}\right)^{-2}\left(X_N + \frac{\bar{N}^2}{A(1+5\times1.25^2)}X_A\right)^{-1}$$

X_N: mass fraction of free nucleons, X_A: mass fraction of nuclei

$\rho_{12} \equiv \frac{\rho}{10^{12}\,g\,cm^{-3}}$, $\bar{N}$: average number of neutrons insid nuclei

Note that neutrinos are produced from capture of electrons by protons. Their energy E_ν is therefore related to the energy of electrons. One can take $\langle E_\nu\rangle \simeq E_{F,e}$ (a precise calculation shows that $\langle E_\nu\rangle = \frac{5}{6}E_{F,e}$).

Taking the core composition to be $X_N=0$, $X_A=1$ with $(A,Z)\simeq {}^{56}Fe$, we obtain:

$$\boxed{\lambda_\nu \simeq 0.3 \times 10^{5}\, \rho_{12}^{-\frac{5}{3}}\ \text{cm}}$$

Assuming a one-dimensional random walk, the number of scatterings N_{scat} before neutrinos escape the core is $N_{scat}^{\frac{1}{2}} \simeq \left(\frac{R}{\lambda_\nu}\right)$.

This results in a diffusion time:

$$\tau_{diff} \simeq N_{scat}\left(\frac{\lambda_\nu}{c}\right) \simeq \frac{R^2}{c\lambda_\nu} \propto \rho$$

Note that neutrinos move at the speed of light essentially because their mass is very tiny ($< eV$). Solving the diffusion equation in three dimensions brings in a factor of $\frac{3}{\pi^2}$, resulting in:

$$\boxed{\tau_{diff} \simeq 0.02\, \rho_{12}\ s}$$

On the other hand, the free-fall collapse time is given by:

$$\tau_{coll} \simeq \left(\frac{8\pi G\rho}{3}\right)^{-\frac{1}{2}} \boxed{= 1.3 \times 10^{-3}\, \rho_{12}^{-\frac{1}{2}}\ s}$$

During the collapse ρ increases, and hence $\frac{\tau_{diff}}{\tau_{coll}}$ becomes larger.

Once σ_{diff} exceed $\sigma_{6 11}$, neutrinos are trapped inside the core and cannot freely escape. This happens at a density given by:

$$\boxed{\rho_{trap} \simeq 1.5 \times 10^{11} \ g \ cm^{-3}}$$

The neutrino density will then increase very fast. The entire physical state of the system made of baryons, leptons and photons can be uniquely specified by three quantities: T, ρ, and Y_e; or, equivalently, by s, ρ, and Y_e ("s" being the entropy density).

Neutrino trapping has the effect of reducing L_ν significantly. The amount of gravitational potential energy released during collapse of the core from an initial radius R_i to R_{nuc} ($R_i \gg R_{nuc}$, R_{nuc} is the radius at which nuclear densities reached) is of the order $\frac{G M_{core}^2}{R_{nuc}}$. For $M_{core} = M_\odot$, we have $R_{nuc} \simeq 12$ km.

The rate of energy release during the collapse is:

$$\frac{\frac{G M_{Core}^2}{R_{nuc}}}{\tau_{coll}} \simeq 10^{57} \text{ erg s}^{-1} \qquad (M_{Core} = M_\odot)$$

The neutrino luminosity follows;

$$L_\nu \simeq \frac{\frac{G M_{Core}^2}{R_{nuc}}}{\tau_{diff}} \simeq \boxed{10^{52} \text{ erg s}^{-1} \qquad (M_{Core} = M_\odot)}$$

We see that the bulk of liberated gravitational energy is actually transformed to other forms of energy inside the core (thermal energy, excitation energy of the nuclear state, etc) rather than being carried away by neutrinos. This suggests that after the neutrino trapping occurs, the collapse proceeds almost adiabatically in the system. We will discuss in more detail next time.